

Chapter 7: Approximate Longitude from a Noon Sight

If we take a number of sights of the sun at regular intervals before and after its crossing of the local meridian, i.e. around the time when it is highest over the horizon, we can plot the altitude of the sun over time. From this graph, we can then determine the top of the curve with an accuracy of perhaps one or two minutes of angle and one or two minutes of time. We have seen in Chapter 6 (Section 6.1) that H_o , the altitude of the sun above the horizon at the time of the sun's crossing of the boat crossing the meridian, gives us directly our Zenith Distance ZD ($90^\circ - H_o$), from which we can calculate our latitude as $ZD + Dec$; $ZD - Dec$; or $Dec - ZD$ depending on the circumstances.

Knowing the time of crossing of the local meridian gives us our longitude: all we need to know, from the Almanac, is the time when the sun crossed the Greenwich Meridian on that day, always within + or - 16 minutes of 12:00 UTC, and compare it with the time when the sun crossed our local meridian. The difference in time can be converted to an arc that represents our longitude, using the rate of 15° per hour for the sun's apparent movement across the sky.

There are a couple of ways to establish the time of the passage of the sun over the local meridian. One is to note the sun's altitude over the horizon at time T_1 before noon, and wait until the sun comes back down to the same altitude at time T_2 after noon. The time of meridian passage is the average between T_1 and T_2 (fig. 7.1)

Another way to obtain the time of meridian passage is to plot the curve of the sun's altitude before and after noon. For any given altitude of the sun, for instance 64° , 65° and 65.5° in the illustration given below, we can estimate when the sun crossed this altitude before and after noon, and take the average time as a good estimate of the time when the sun reached its highest altitude. The average of those three or four averages provides our best estimate of the time when the sun crossed our meridian (fig. 7.2)

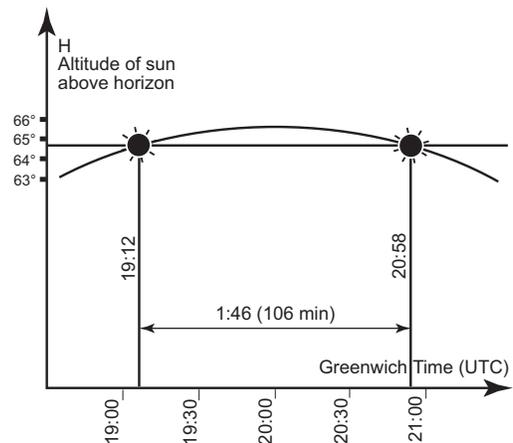


Fig. 7.1. The time when the sun crosses the meridian (local noon) can be established from two measures, at times T_1 and T_2 , when the sun is at the same height (altitude) above the horizon, before and after noon: $T_1 = 19:12$ UTC; $T_2 = 20:58$ UTC; $T = T_2 - T_1 = 106$ min (1 h 46 min) Time of crossing of meridian: $19:12 + 1/2 (106) = 19:12 + 53 = 20:05$ UTC.

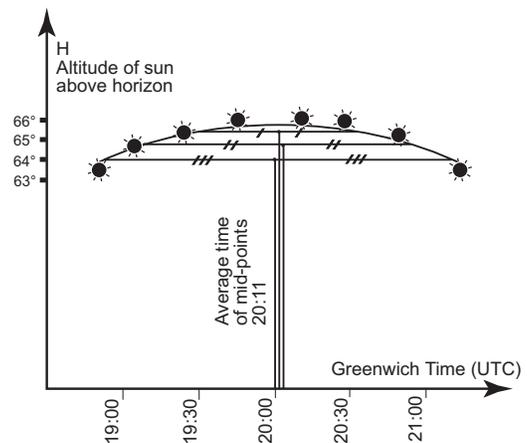


Fig. 7.2. Another way to measure the time when the sun crosses the meridian is to plot the sun's trajectory around noon, and take averages of the times when the sun had the same altitude, before and after noon.

7.1 Example 1: West Longitude

From our plot of the sun before and after noon (see fig. 7.2), we find that the sun reached its maximum altitude at approximately 20:11 UTC. It is approximately noon where we are, but already past 8 p.m. on the meridian of Greenwich: our longitude is therefore west. On what meridian are we?

Let us assume that, in the Almanac for that day, we find that the true sun crossed the Greenwich Meridian at 12:06 UTC. This means that the sun crosses each meridian at 12:06 on that day, and in particular the boat meridian.

Time difference between the sun crossings of the Greenwich and boat meridians:

$$\begin{array}{r}
 \text{Time difference} \quad 20:11 \\
 \quad \quad \quad \quad \quad - 12:06 \\
 \hline \hline
 \quad \quad \quad \quad \quad 08:05
 \end{array}$$

The true sun has thus taken 8 hours and 05 minutes to travel from the Greenwich Meridian to the meridian where our boat is, at 15° per hour.

In order to determine our longitude, we simply need to multiply the time difference between the meridian crossings, 8 h 05 min, by $15^\circ/\text{h}$.

Considering the hours first, our longitude is $15^\circ/\text{h} \times 8 \text{ h} = 120^\circ$ west of Greenwich. Alternatively, we could use the table *Conversions of Arc to Time* in Appendix 2, p. 15 (column $120^\circ - 179^\circ$, first line).

For the 05 minutes, we look at the *Almanac Increments and Corrections Table* (Appendix 2, p. 16):

In 05 min, the sun travels $01^\circ 15.0'$. Our longitude is therefore:

$$120^\circ + 01^\circ 15.0' = 121^\circ 15.0' \text{ W}$$

7.2 Example 2: East Longitude

From another plot of the sun before and after noon, another day, we now find for example that the sun reached its maximum altitude at 10:08 UTC. It is approximately noon on our meridian, but the sun will need to travel another two hours before reaching the meridian of Greenwich; our longitude is therefore east.

First, we find at what time the true sun will cross the Greenwich Meridian. In the Almanac, for that day, we find for example the crossing at 11:53.

$$\begin{array}{r} \text{Time difference} \quad 11:53 \\ \quad \quad \quad \quad - 10:08 \\ \hline \quad \quad \quad \quad 01:45 \end{array}$$

At 15° per hour, the sun will take 1 h and 45 min to travel from the meridian of the boat to the Greenwich Meridian. For one hour of time difference, at 15° per hour, our longitude is:

$$015^\circ/\text{h} \times 1 \text{ h} = 015^\circ 00' \text{ East of Greenwich}$$

For the 45 minutes, we find $011^\circ 15'$ in the Almanac, *Increments and Conversions* in Appendix 2, p. 23. Our longitude is therefore:

$$015^\circ 00' + 011^\circ 15.0' = 026^\circ 15' \text{ E}$$

7.3 Accuracy of Longitude Estimates from a Noon Sight

This way of establishing our longitude is not very precise: it is very difficult to determine the exact moment when the sun reaches its maximum altitude over the horizon because the top of the curve is fairly flat: an accuracy of one or two minutes of time might be achieved at best.

The apparent rate of revolution of the sun is 15° per hour, or 1° of angle in four minutes of time. This means that a difference in time of four minutes in the measurement of noon passage translates into a difference of one degree in longitude, or 60 miles at the equator. An error of only one minute in time represents 15 miles of error in longitude at the equator; the error gets gradually smaller as one moves away from the equator towards the N or S poles. Unless a great precision in longitude is required, the simple method described above can still be very useful.

With an Almanac to determine the Declination of the sun on that day, but without any Sight Reduction tables or plotting sheets, a navigator can thus take a series of sun sights before and after noon, plot them, and determine the boat latitude with a high degree of precision, typically within one or two minutes of angle, i.e. within one or two miles (Chapter 6). Again, with an Almanac to determine at what time the sun crosses each meridian on that day, but without Sight Reduction tables, a noon sight provides an indication of longitude if we have an accurate timepiece set on the Greenwich time. The accuracy of the longitude thus calculated is, however, rather poor and typically within half a degree at best, or 30 miles at the equator.